Connecting the Concepts

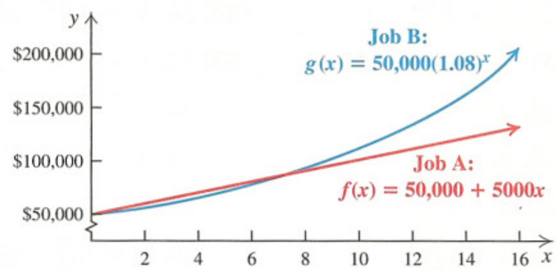
Linear and Exponential Growth

Linear functions increase (or decrease) by a constant amount. The rate of change (slope) of a linear graph is constant. The rate of change of an exponential graph is not constant.

To illustrate the difference between linear growth and exponential growth, compare the salaries for two hypothetical jobs, shown in the table and the graph below. Note that the starting salaries are the same, but the salary for job A increases by a constant amount (linearly) and the salary for job B increases by a constant percent (exponentially).

Note that the salary for job A is larger for the first few years only. The rate of change of the salary for job B increases each year, since the increase is based on a larger salary each year.

	Job A	Job B
Starting salary	\$50,000	\$50,000
Guaranteed raise	\$5000 per year	8% per year
Salary function	$f(x) = 50{,}000 + 5000x$	$g(x) = 50{,}000(1.08)^x$



9.2 Exercise Set

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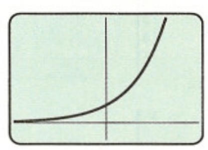
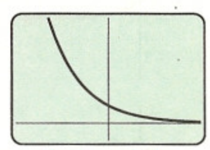
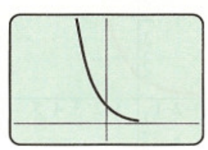
Concept Reinforcement *Classify each of the following statements as either true or false.*

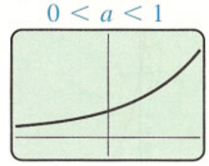
1. The graph of $f(x) = a^x$ always passes through the point $(0, 1)$. True
2. The graph of $g(x) = \left(\frac{1}{2}\right)^x$ gets closer and closer to the x-axis as x gets larger and larger. True
3. The graph of $f(x) = 2^{x-3}$ looks just like the graph of $y = 2^x$, but it is translated 3 units to the right. True
4. The graph of $g(x) = 2^x - 3$ looks just like the graph of $y = 2^x$, but it is translated 3 units up. False
5. The graph of $y = 3^x$ gets close to, but never touches, the y-axis. False
6. The graph of $x = 3^y$ gets close to, but never touches, the y-axis. True

Each of Exercises 7–10 shows the graph of a function $f(x) = a^x$. Determine from the graph whether $a > 1$ or $0 < a < 1$.

7.  $a > 1$

8.  $0 < a < 1$

9.  $0 < a < 1$

10.  $a > 1$

Graph.

11. $y = f(x) = 3^x$ ⊡

12. $y = f(x) = 4^x$ ⊡

13. $y = 5^x$ ⊡

14. $y = 6^x$ ⊡

15. $y = 2^x + 3$ ⊡

16. $y = 2^x + 1$ ⊡

17. $y = 3^x - 1$ ⊡

18. $y = 3^x - 2$ ⊡

19. $y = 2^{x-3}$ ⊡

20. $y = 2^{x-1}$ ⊡

21. $y = 2^{x+3}$ ⊡

22. $y = 2^{x+1}$ ⊡

23. $y = \left(\frac{1}{5}\right)^x$ ⊡

24. $y = \left(\frac{1}{4}\right)^x$ ⊡

25. $y = \left(\frac{1}{10}\right)^x$ ⊡

26. $y = \left(\frac{1}{3}\right)^x$ ⊡

27. $y = 2^{x-3} - 1$ ⊡

28. $y = 2^{x+1} - 3$ ⊡

29. $y = 1.7^x$ ⊡

30. $y = 4.8^x$ ⊡

31. $y = 0.15^x$ ⊡

32. $y = 0.98^x$ ⊡

33. $x = 3^y$ ⊡

34. $x = 6^y$ ⊡

35. $x = 2^{-y}$ ⊡

36. $x = 3^{-y}$ ⊡

37. $x = 5^y$ ⊡

38. $x = 4^y$ ⊡

39. $x = \left(\frac{3}{2}\right)^y$ ⊡

40. $x = \left(\frac{4}{3}\right)^y$ ⊡

Graph each pair of equations using the same set of axes.

41. $y = 3^x,\ x = 3^y$ ⊡

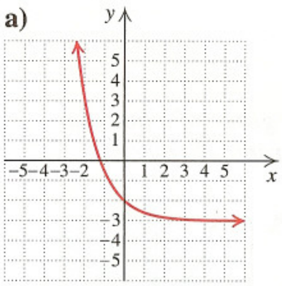
42. $y = 4^x,\ x = 4^y$ ⊡

43. $y = \left(\frac{1}{2}\right)^x,\ x = \left(\frac{1}{2}\right)^y$ ⊡

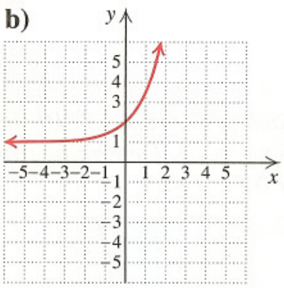
44. $y = \left(\frac{1}{4}\right)^x,\ x = \left(\frac{1}{4}\right)^y$ ⊡

Aha! *In Exercises 45–50, match each equation with one of the following graphs.*

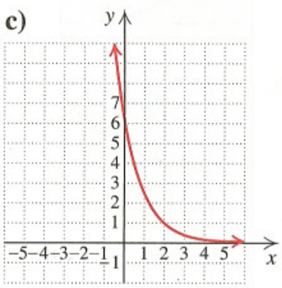
a)



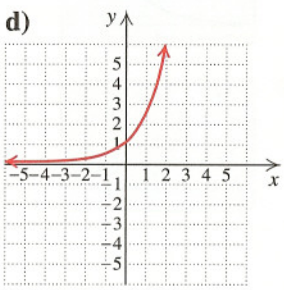
b)



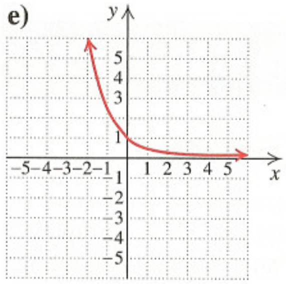
c)



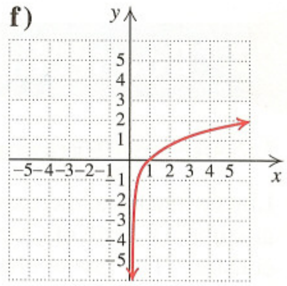
d)



e)



f)



45. $y = \left(\frac{5}{2}\right)^x$ (d)

46. $y = \left(\frac{2}{5}\right)^x$ (e)

47. $x = \left(\frac{5}{2}\right)^y$ (f)

48. $y = \left(\frac{2}{5}\right)^x - 3$ (a)

49. $y = \left(\frac{2}{5}\right)^{x-2}$ (c)

50. $y = \left(\frac{5}{2}\right)^x + 1$ (b)

Solve.

51. *Music Downloads.* The number $M(t)$ of single tracks downloaded, in billions, t years after 2003 can be approximated by

$$M(t) = 0.353(1.244)^t.$$

Source: Based on data from International Federation of the Phonographic Industry

a) Estimate the number of single tracks downloaded in 2006, in 2008, and in 2012. ⊡

b) Graph the function. ⊡

52. *Growth of Bacteria.* The bacteria *Escherichia coli* are commonly found in the human bladder. Suppose that 3000 of the bacteria are present at time $t = 0$. Then t minutes later, the number of bacteria present can be approximated by

$$N(t) = 3000(2)^{t/20}.$$

a) How many bacteria will be present after 10 min? 20 min? 30 min? 40 min? 60 min? ⊡

b) Graph the function. ⊡

53. *Smoking Cessation.* The percent of smokers $P(t)$ who, with telephone counseling to quit smoking, are still successful t months later can be approximated by

$$P(t) = 21.4(0.914)^t.$$

Sources: *New England Journal of Medicine*; data from California's Smokers' Hotline 19.6%; 16.3%; 7.3%

a) Estimate the percent of smokers receiving telephone counseling who are successful in quitting for 1 month, 3 months, and 1 year.

b) Graph the function. ⊡

⊡ Answers to Exercises 11–44, 51, 52, and 53(b) are on p. IA-23.

54. *Smoking Cessation.* The percent of smokers $P(t)$ who, without telephone counseling, have successfully quit smoking for t months (see Exercise 53) can be approximated by

$$P(t) = 9.02(0.93)^t.$$

Sources: *New England Journal of Medicine*; data from California's Smokers' Hotline

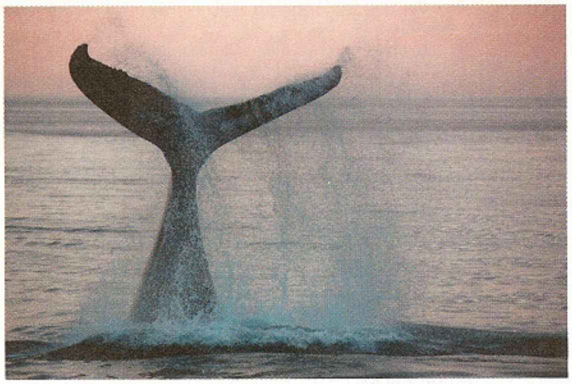
a) Estimate the percent of smokers not receiving telephone counseling who are successful in quitting for 1 month, 3 months, and 1 year. 8.4%; 7.3%; 3.8%

b) Graph the function. ⊡

55. *Marine Biology.* Due to excessive whaling prior to the mid 1970s, the humpback whale is considered an endangered species. The worldwide population of humpbacks $P(t)$, in thousands, t years after 1900 $(t < 70)$ can be approximated by

$$P(t) = 150(0.960)^t.$$

Source: Based on information from the American Cetacean Society, 2006, and the ASK Archive, 1998



a) How many humpback whales were alive in 1930? in 1960? About 44,079 whales; about 12,953 whales

b) Graph the function. ⊡

56. *Salvage Value.* A laser printer is purchased for \$1200. Its value each year is about 80% of the value of the preceding year. Its value, in dollars, after t years is given by the exponential function

$$V(t) = 1200(0.8)^t.$$

a) Find the value of the printer after 0 year, 1 year, 2 years, 5 years, and 10 years. ⊡

b) Graph the function. ⊡

57. *Marine Biology.* As a result of preservation efforts in most countries in which whaling was common, the humpback whale population has grown since the 1970s. The worldwide population of humpbacks $P(t)$, in thousands, t years after 1982 can be approximated by

$$P(t) = 5.5(1.08)^t.$$

Source: Based on information from the American Cetacean Society, 2006, and the ASK Archive, 1998

a) How many humpback whales were alive in 1992? in 2006? About 11,874 whales; about 34,876 whales

b) Graph the function. ⊡

58. *Recycling Aluminum Cans.* About $\frac{1}{2}$ of all aluminum cans will be recycled. A beverage company distributes 250,000 cans. The number in use after t years is given by the exponential function

$$N(t) = 250{,}000\left(\tfrac{1}{2}\right)^t.$$

Source: The Aluminum Association, Inc., 2009

a) How many cans are still in use after 0 year? 1 year? 4 years? 10 years? 250,000; 125,000; 15,625; 244

b) Graph the function. ⊡

59. *Invasive Species.* Ruffe is a species of freshwater fish that is considered invasive where it is not native. The function

$$R(t) = 2(1.75)^t$$

can be used to estimate the number of ruffe in a lake t years after 2 fish have been introduced to the lake.

Source: Based on information from invasivespeciesireland.com

a) How many fish will be in the lake 10 years after 2 ruffe have been introduced? after 15 years? ⊡

b) Graph the function. ⊡

60. *mp3 Players.* The number $m(t)$ of mp3 players sold per year in the United States, in millions, can be estimated by

$$m(t) = 34.3(1.25)^t,$$

where t is the number of years after 2006.

Source: Forrester Research, Inc.

a) Estimate the number of mp3 players sold in 2006, in 2010, and in 2012. ⊡

b) Graph the function. ⊡

TW **61.** Without using a calculator, explain why 2^π must be greater than 8 but less than 16.

TW **62.** Suppose that \$1000 is invested for 5 years at 7% interest, compounded annually. In what year will the most interest be earned? Why?

SKILL REVIEW

Review factoring polynomials (Sections 5.3–5.7).

Factor.

63. $3x^2 - 48$ [5.6] $3(x + 4)(x - 4)$

64. $x^2 - 20x + 100$ [5.6] $(x - 10)^2$

65. $6x^2 + x - 12$ [5.5] $(2x + 3)(3x - 4)$

66. $8x^6 - 64y^6$ [5.7] $8(x^2 - 2y^2)(x^4 + 2x^2y^2 + 4y^4)$

67. $6y^2 + 36y - 240$ [5.4] $6(y - 4)(y + 10)$

68. $5x^4 - 10x^3 - 3x^2 + 6x$ [5.3] $x(x - 2)(5x^2 - 3)$

⊡ Answers to Exercises 54(b), 55(b), 56, 57(b), 58(b), 59, and 60 are on p. IA-23.

SYNTHESIS

TW **69.** Examine Exercise 60. Do you believe that the equation for the number of mp3 players sold in the United States will be accurate 20 years from now? Why or why not?

TW **70.** Why was it necessary to discuss irrational exponents before graphing exponential functions?

Determine which of the two given numbers is larger. Do not use a calculator.

71. $\pi^{1.3}$ or $\pi^{2.4}$ $\pi^{2.4}$

72. $\sqrt{8^3}$ or $8^{\sqrt{3}}$ $8^{\sqrt{3}}$

Graph.

73. $y = 2^x + 2^{-x}$ ⊡

74. $y = \left|\left(\frac{1}{2}\right)^x - 1\right|$ ⊡

75. $y = |2^x - 2|$ ⊡

76. $y = 2^{-(x-1)^2}$ ⊡

77. $y = |2^{x^2} - 1|$ ⊡

78. $y = 3^x + 3^{-x}$ ⊡

Graph both equations using the same set of axes.

79. $y = 3^{-(x-1)}$, $x = 3^{-(y-1)}$ ⊡

80. $y = 1^x$, $x = 1^y$ ⊡

81. *Navigational Devices.* The number of GPS navigational devices in use in the United States has grown from 0.5 million in 2000 to 4 million in 2004 to 50 million in 2008. After pressing STAT and entering the data, use the ExpReg option in the STAT CALC menu to find an exponential function that models the number of navigational devices in use t years after 2000. Then use that function to predict the total number of devices in use in 2012. $N(t) = 0.464(1.778)^t$; about 464 million devices

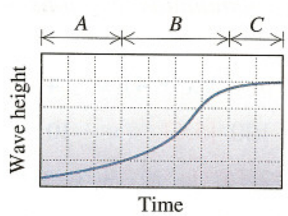
Source: Telematics Research Group

82. *Keyboarding Speed.* Trey is studying keyboarding. After he has studied for t hours, Trey's speed, in words per minute, is given by the exponential function

$$S(t) = 200[1 - (0.99)^t].$$

Use a graph and/or table of values to predict Trey's speed after studying for 10 hr, 40 hr, and 80 hr. 19 wpm, 66 wpm, 110 wpm

TW **83.** The graph below shows growth in the height of ocean waves over time, assuming a steady surface wind.
Source: magicseaweed.com



Source: magicseaweed.com

a) Consider the portions of the graph marked A, B, and C. Suppose that each portion can be labeled Exponential Growth, Linear Growth, or Saturation. How would you label each portion?

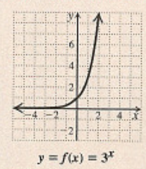
b) Small vertical movements in wind, surface roughness of water, and gravity are three forces that create waves. How might these forces be related to the shape of the wave-height graph?

TW **84.** Consider any exponential function of the form $f(x) = a^x$ with $a > 1$. Will it always follow that $f(3) - f(2) > f(2) - f(1)$, and, in general, $f(n + 2) - f(n + 1) > f(n + 1) - f(n)$? Why or why not? (*Hint*: Think graphically.)

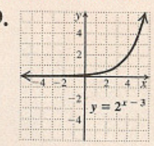
85. On many graphing calculators, it is possible to enter and graph $y_1 = A\wedge(X - B) + C$ after first pressing APPS Transfrm. Use this application to graph $f(x) = 2.5^{x-3} + 2$, $g(x) = 2.5^{x+3} + 2$, $h(x) = 2.5^{x-3} - 2$, and $k(x) = 2.5^{x+3} - 2$.

Try Exercise Answers: Section 9.2

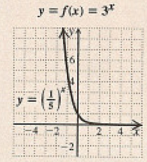
11.



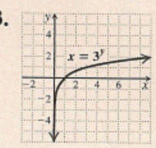
19.



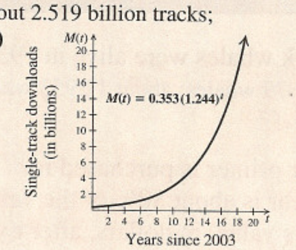
23.



33.



51. **(a)** About 0.68 billion tracks; about 1.052 billion tracks; about 2.519 billion tracks;
(b)



⊡ Answers to Exercises 73–80 are on p. IA-24.